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# IMPLEMENTATION OF A FULL DOPPLER COMPENSATION TO ELIMINATE THE USE OF FIRST ORDER APPROXIMATION TO COMPENSATE THE DOPPLER IN LARGE TIME-BANDWIDTH PRODUCT RADAR IMAGES

by

## PAVAN KUMAR VUTUKUR

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science Department of Electrical Engineering

Dr. Hector A. Ochoa, Ph.D., Committee Chair

College of Engineering and Computer Science

The University of Texas at Tyler August 2011 The University of Texas at Tyler Tyler, Texas

This is to certify that the Master's Thesis of

#### PAVAN KUMAR VUTUKUR

has been approved for the thesis requirement on April 29, 2011 for the Master of Science Degree in Electrical Engineering

Approvals:

Thesis Chair: Hector A. Ochoa, Ph.D.

Member: Hassan El-Kishky, Ph.D.

rud

Member: David Hoe, Ph.D.

Chair and Graduate Coordinator: Mukul V. Shirvaikar Ph.D.

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James K. Nelson, Jr., Ph.D., P.E., Dean, College of Engineering and Computer Science, Brazzel Professor of Engineering

In memory of my Grandfather Late "Shri" Anand T Vutukur

#### Acknowledgements

Much of the work in this thesis originated through the prior Ph.D. dissertation of Dr. Hector A Ochoa. As my thesis advisor, I have enjoyed working under him. His suggestions and constructive comments enabled me to remain motivated to go that extra mile in my research. I would like to specifically acknowledge the contribution made in this field of research by Dr. Benjamin Flores, Professor and Acting Dean of Graduate School, UT El Paso.

I am deeply indebted to my committee members Dr. Hassan El-Kishky and Dr. David Hoe for their time in reviewing my work and providing valuable comments and suggestions. Additional thanks to Dr. Mukul V. Shirvaikar, Chair, Department of Electrical Engineering, for his time in reviewing my thesis and for the constant support throughout my graduate program.

In conclusion, I would like to thank my family, relatives and friends who have contributed directly or indirectly in realizing my goals.

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### Abstract

# IMPLEMENTATION OF A FULL DOPPLER COMPENSATION TO ELIMINATE THE USE OF FIRST ORDER APPROXIMATION TO COMPENSATE THE DOPPLER IN LARGE TIME-BANDWIDTH PRODUCT RADAR IMAGES

Pavan Kumar Vutukur Thesis Chair: Hector A. Ochoa, Ph.D.

The University of Texas at Tyler August 2011

The first order approximation technique is used in current generation of radars to compensate the frequency shift due to the target's velocity. A common practice in radar systems to improve their resolution and performance is to increment the time-bandwidth (TBW) product of the transmitted signal. As the first order approximation is comprised of only the first two terms of a power series expansion, any significant increase in the TBW product of the signal results in significant errors in the radar image. The proposed method in this research gives a relativistic analysis of the scattered signal to create a full Doppler compensation and reduce the errors induced by the target's velocity.

In the first stage of this research, a linear frequency modulated (chirp) signal with a large TBW is considered as the transmitted signal. First, to observe the effects of increasing the time-bandwidth product and the target's velocity, the received signal is modeled using the full Doppler compensation and the first order approximation. Second, each signal is applied to the input of a matched-filter in which the transmitted signal is used as a reference. Finally, the outputs from both matched-filters are analyzed in order to observe the effects of using the first order approximation to model the Doppler effect induced on the reflected signal. The analysis was performed assuming that the target was moving at a constant velocity. By increasing the time-bandwidth product of the transmitted signal, the output of both matched-filters are compared and analyzed to observe the differences between modeling the reflected signal using the first order compensation and the full Doppler compensation. The simulation results have shown that by increasing the time-bandwidth product of the transmitted signal, the output of the matched-filter using the first order approximation deviates significantly with respect to the output of the matched-filter that contains the signal modeled using a full Doppler compensation.

In the second stage, a radar image was generated by applying Fourier transformation techniques to the target's reflected signal. The generated image is now separately compensated using the first order approximation and the full Doppler compensation techniques and auto-correlated with a match-filter. The simulation results obtained after the matched-filter have shown a considerable deviation in position and a significant distortion in the resolution of the reflected image when the first order approximation technique is used. From these results it is concluded that a dramatic increase in the time-bandwidth product of the received signal leads to a significant error at the output of the matched-filter if the first order approximation is used to model the reflected signal instead of the full Doppler compensation.

## **Chapter One**

## Introduction

The freedom to design various characteristics into the radar signal has been an important factor in the development of modern radar systems. The basic design criterion in modern radars is to enhance the target detection and the parameter estimation, by ensuring that the received signals contain sufficient energy to be visible within the noise. An adequate implementation of the matched-filter and pulse-compression is necessary to overcome such practical problems and measurement challenges. High resolution and increased system resolving capabilities place special demands in terms of bandwidth and phase, which are some of the basic challenges in modern radar systems [1]. The pulse Doppler radar, also known as MTI Radar (Moving Target Indicator Radar), employs the Doppler shift for the detection of moving targets [2]. The evolution of pulsed Doppler radar led to the integration of waveform design into radar systems

A typical example used to describe the Doppler effect is the change of pitch of an audible sound signal originated from a locomotive as it travels toward or away from the listener. Similarly, in this case the Doppler effect changes the frequency of the electromagnetic signal that propagates from the radar to the moving target and back to the radar.

The linear frequency-modulated (Linear FM) signal or chirp signal is an electromagnetic waveform used in pulse Doppler radars. A linear FM signal consists of a rectangular pulse of constant amplitude and duration. The frequency of this signal increases linearly from an initial frequency ( $f_1$ ) to a final frequency ( $f_2$ ) over the duration of the pulse. At the receiver side, the frequency modulated signal is passed through a matched-filter which speeds up the higher frequencies at the trailing edge of the

pulse relative to the lower frequencies at the leading edge. The signal from the matchedfilter will be compressed to a width 1/B where  $B = f_2 - f_1$ . The signal when passed through the filter has its pulse peak increased by the pulse compression ratio  $B \times T = TBW$ , known as the time-bandwidth Product [2].

The received signal is analyzed by assuming that there is no signal interference with a target resolution process that assists the receiver system to deduce the characteristics of the targets that are detected on the radar. As a result, it has become necessary to implement a matched-filter receiver to optimize the waveform and reduce the mutual interference between targets, to the point where it becomes negligible. As stated by Woodward [3], the envelope of the matched-filter response gives the measured probability distribution for the target range. In current generation radars, the received signal at the matched-filter is compensated using a first order approximation technique. This technique becomes unsuitable when large time-bandwidth product signals are used to improve the resolution of the radar image. For that reason, the aim of this research is to explore the effects caused by using the first order approximation to compensate the Doppler in large time-bandwidth product signals and at the same time proposes the use of a full Doppler compensation technique in this regard.

#### **Review of prior research**

The recent work by Boehm III and Debroux [4] discussed the effects of a moving target on the received signal and the ambiguity function using a Doppler invariant signal. By applying the principles of special relativity, the effects on the received signal due to modifications created by the observers and relative motion were determined using Poincare transformations. The waveform was then compensated so that the output has no distortion with respect to the input signal. Additional work by Ochoa and Flores [5], Ochoa and Nakka [6] addressed the implementation of a first order compensation technique in the matched-filter as a Fourier kernel to improve the resolution of the received signal in a radar system. It was also shown that under relativistic speed targets, this technique becomes inadequate leading to distortions in the signals. Instead, a full Doppler compensation technique was proposed to avoid frequency mismatches and power losses, allowing an accurate estimation of range and velocity.

These results served as the foundation for this research. The rationale is to expand the research to include the full Doppler compensation technique to improve the resolution of high velocity targets when large time-bandwidth products signals are applied. Having simulated this technique [7] for large time-bandwidth product signals under high velocity conditions, the compensated signals from the matched-filter showed a significant mismatch between the two techniques on a step by step increment of the Time-Bandwidth product of the received signal. In addition to this, it was also observed that there exists a linear dependency between the increase of the error produced by using the first order approximation and the time-bandwidth product.

This research extends a similar approach used for a received radar signal to generate a radar image by applying Fourier transformation techniques. After applying both compensation techniques independently, a radar image is generated from the matched-filter output to compare the results from each technique.

#### **Research objective**

The goal of the work is to show the inadequacy of using a first order approximation technique under a large target velocity and an increment in the timebandwidth product of the signal. A thorough literature review has shown that most of the researchers have pointed out the distortion in using the first order approximation technique. It was also suggested that the full Doppler compensation technique is a viable alternative in maintaining the accuracy of the received signals under higher resolutions. By applying the above mentioned conditions the focus of this research intends not only on emphasizing the use of full Doppler compensation technique, but also to extend this technique to the received radar image.

#### **Organization of thesis**

This thesis is divided into four chapters. Chapter 2 discusses the chirp signal and the Ambiguity Function in the compensation techniques used in this research. It also gives an overview of the Doppler Effect and the present application in Doppler Radar Systems. Chapter 3 describes the two compensation techniques in detail and the signals received from the matched-filter upon processing. The same technique is applied to generate a radar image from a matched-filter by using the above compensation methods.

Chapter 4 includes conclusions and some examples of the future work that could be performed.

# **Chapter Two**

## Background

Radar signals are designed to meet certain spectral and Doppler tolerance characteristics for range detection and resolution. An example of this is the linear frequency modulated signal. The reason for choosing this signal is due to the fact that its instantaneous frequency varies linearly with time, making the rate of frequency sweep constant [3].

### Linear frequency modulated signal

A linear frequency modulated signal is often referred to as the "chirp" signal due to the sound effect of a linearly increasing instantaneous frequency. The term *instantaneous frequency* refers to the *time-dependent frequency* of a signal whose complex expression for a narrow band signal (small fractional bandwidth) can be written as

$$s(t) = a(t)e^{j\psi(t)}$$
(2.1)

where a(t) is the amplitude of the modulation envelope and  $\Psi(t)$  is the signal phase defined as  $2\pi f_c t + \theta(t)$ , where t is the time,  $f_c$  is the carrier frequency and  $\theta(t)$  is the phase modulation function. The amplitude modulation term a(t) for small fractional bandwidth does not affect the instantaneous frequency [8]. The instantaneous frequency of the signal is defined as

$$f_i = \frac{1}{2\pi} \frac{d\psi}{dt} \tag{2.2}$$

In order to illustrate the concept of instantaneous frequency, the echo from a target as seen by a single-frequency pulsed-Doppler radar is considered. The Doppler frequency is given by equation

$$f_d = \frac{2vf_c}{c} \tag{2.3}$$

where v is the target's radial velocity towards the radar, c is the propagation velocity and  $f_c$  is the transmitted carrier frequency. The instantaneous phase advance of the echo signal from the target at a time t is given by

$$\psi(t) = \frac{4\pi f_c}{c} \left(R - vt\right) \tag{2.4}$$

where R is the target range at time t = 0, v is the constant velocity toward the radar at t = 0. The complex form of the echo signal observed with the radar transmitting at a frequency  $f_c$  is given by

$$s(t) = a(t) \exp \left[ 2\pi f_c t - \frac{4\pi f_c t}{c} \left( R - vt \right) \right]$$
(2.5)

In a typical arrangement, a(t) defines the envelope of a train of fixed frequency transmitted pulses. For pulse-compression radar, the transmitted Chirp pulse contains a quadratic-phase term by design. Therefore, a linear change of frequency (called linear FM) exists on each echo pulse, even when the target is stationary. Each echo pulse contains the designed linear FM, which greatly exceeds pulse caused by target's acceleration. Thus the linear FM signal becomes easy to process as the requirement for using a large Doppler filter bank is not necessary. As a result, a single filter is adequate as the complexity of the correlator is reduced since a single time setting or channel is adequate.

#### **Matched-filter**

The matched-filter is commonly used in radar in which it maximizes the output peak-SNR and matches to the spectrum of the signal expected at a particular Doppler shift  $f_d$ . This makes the matched-filter a basis of design of all radar receivers.

The matched-filter characteristics can be designated by either a frequency response function or a time response function, each of them related by a Fourier transform. In the frequency domain, the matched-filter transfer function  $H(\omega)$  is the complex conjugate function of the spectrum of the signal that is to be processed in an optimum fashion. Thus, the equation for the transfer function of a matched-filter with an input signal s(t) with spectrum  $S(\omega)$  is defined as

$$H(\omega) = GS^*(\omega) \exp\left[-j\omega T_d\right]$$
(2.6)

and

$$h(t) = Gs^{*}(T_{d} - t)$$
(2.7)

where  $T_d$  is the delay constant required to make the filter physically reliable and G is the fixed component of the net gain through the filter. For this research, the transmitted linear FM signal is modeled as [9]

$$s(t) = e^{j\{(2\pi t) + (\pi t^2)\}} \left[ U(t) - U(t - T) \right]$$
(2.8)

where *k* represents the frequency sweep of the transmitted signal and *f* represents the frequency of the transmitted signal. The equation of the reflected signal from a moving target can be modeled from equation 2.7 and 2.8 as follows [7]

$$s_r(t) = \alpha s(\alpha t) \left[ U(\alpha t) - U(\alpha t - T) \right]$$
(2.9)

where  $\alpha$  represents the compensation technique used to model the reflected signal. This term can be either the typically known first order approximation or a full Doppler Compensation [5][9]. Both of these terms will be discussed in detail in the following chapter (Chapter 3).

The matched-filter processing of an echo signal is a coherent summation of the reflected signal from the target's reflection points which are spread over the target's range. Regardless of the target, in the matched-filter, the match is made for the transmitted waveform, which remains constant. A well known matched-filter for high-resolution radar is the pulse-compression filter.

#### Signal to noise ratio (SNR) and time-bandwidth product

The optimum filter for detection of a signal is the matched-filter, which has a frequency response  $H(\omega)$  equal to the complex conjugate of the signal spectrum, and an impulse response equal to the time-reversed conjugate of the signal waveform. Wehner[8] and North[10] have discussed that the ratio of the output response of the matched-filter Signal to the Average Noise (SNR) Ratio is equal to twice the signal energy *E* (received) over noise power per hertz  $N_0$ . This expression is given as

$$\left(\frac{\hat{S}}{N}\right)_{out} = \frac{2E}{N_0} \tag{2.10}$$

From equation 2.10, it has to be understood that the noise power per hertz in terms of a two sided spectrum with positive and negative frequencies is  $N_0/2$ . From equation 2.10, the SNR over pulse duration  $T_1$  can be given as

$$\left(\frac{\hat{S}}{N}\right)_{out} = \frac{2ST_1}{N/\beta_n}$$
(2.11)

where *N* is the input noise power of the matched-filter receiver with a noise bandwidth  $\beta_n$ . Additionally, *S* and  $\hat{S}$  refer to the average input power and peak instantaneous signal power respectively. From equations 2.10 and 2.11 the relationship between the input peak SNR and output peak SNR is given as

$$\left(\frac{S}{N}\right)_{in} = \frac{1}{2T_1\beta_n} \left(\frac{\hat{S}}{N}\right)_{out}$$
(2.12)

The quantity  $T_1\beta_n$  in Equation 2.12 is the time-bandwidth (TBW) product of a signal. It can be made greater than unity by frequency or phase coding. For ideal matching, the ratio of the SNR output over the input (signal processing gain) approaches the time-bandwidth product.

### **Ambiguity function**

The concept of ambiguity function reveals the waveform's inherent performance in the aspect of the range and Doppler resolution. From Equation 2.8 the ambiguity function of the linear FM Chirp signal s(t) is defined as the cross-correlation of the Doppler unshifted waveform with the waveform that has not been shifted. From the definition of cross-correlation [11]

$$\chi(\tau, f_D) = \int_{-\infty}^{\infty} \left[ s(t) e^{2\pi f_D t} \left[ s^*(t+\tau) \right] dt$$
(2.13)

where  $s(t)e^{2\pi f_D t}$  is the Doppler-shifted version of the waveform s(t). By re-arranging the terms in equation 2.13 the following equation is obtained

$$\chi(\tau, f_D) = \int_{-\infty}^{\infty} s(t) s^*(t+\tau) e^{2\pi f_D t} dt$$
(2.14)

Equation 2.14 produces a common form of the ambiguity function as  $|\chi(\tau, f_D)|^2$ and the shape of the waveform is entirely dependent of the waveform parameters.

# **Chapter Three**

# First order approximation and full Doppler compensation techniques

In order to improve the resolution of the received signal, a Doppler compensation is applied to the reflected signal at the input of the matched-filter. In this aspect, typically, the first order approximation is used to model the effects induced by the target's velocity. Usually, the Doppler effect is modeled as the frequency shift proportional to the target's velocity [4] [5]. This is done by using the following expression

$$\alpha_1 = 1 - 2\left(\frac{\nu}{c}\right) \tag{3.1}$$

where *v* is the velocity of the target and *c* is the speed of light. The implementation of a first order approximation is acceptable when low-velocity targets ( $v \ll c$ ) and small time-bandwidth product signals are considered. Equation 3.1 is obtained by extracting the first two terms of a power series expansion and by neglecting all the higher order terms. On the other hand, the full Doppler compensation is defined as

$$\alpha_2 = \left(\frac{c-v}{c+v}\right) \tag{3.2}$$

where c corresponds to the speed of light and v is the target's velocity. If the full Doppler expression from equation 3.2 is expanded using power series, it is clear how the first two terms of the expansion corresponds to the first order approximation from equation 3.1

$$\alpha_2 = 1 - 2\frac{v}{c} + 2\frac{v^2}{c^2} - 2\frac{v^3}{c^3} + \dots$$
(3.3)

From equation 3.3 it can be observed that the first order approximation can be obtained by extracting the first two terms of a power series expansion and by neglecting all higher order terms. For this reason, the first order approximation is inadequate when high velocity targets or large time-bandwidth product signals are considered. In order to eliminate the error produced by the higher order terms a full Doppler compensation should be considered to model the reflected signal. By using equations 2.8 and 2.9, the reflected signal can be modeled using the compensation factor  $\alpha$  as shown below

$$s(\alpha t) = e^{j\{(2\pi f \alpha t) + (\pi k \alpha^2 t^2)\}} [U(\alpha t) - U(\alpha t - T)]$$
(3.4)

If it is desired to model the received signal using the first order approximation the variable  $\alpha$  needs to be replaced by the expression in equation 3.1. On the other hand, if the full Doppler compensation is desired  $\alpha$  will need to be replaced by the expression shown in equation 3.2. Equation 3.4 is modeled as the reflected signal and is compensated to send it as an input to the matched-filter to simulate the compensation techniques.

#### Doppler compensation of a received signal

In order to observe the effects of using a first order approximation to compensate the Doppler induced by the target's velocity, a linear frequency modulated signal is considered as the transmitted signal. The reflected signal is modeled using equation 3.4. In order to compensate the reflected signal, a first order approximation and full Doppler compensation is applied. The parameters of the transmitted signal used in the simulation are shown in Table 1[7]

Initial Frequency	25 GHz
Bandwidth	0.5 GHz
Initial TBW Product	250
Final TBW Product	5000

Table 1. Parameters used to simulate the transmitted linear FM signal

By using these parameters the duration of the transmitted signal could be easily calculated using the following expression

$$T = \frac{TBW}{B} \tag{3.5}$$

where TBW represents the time-bandwidth product and B represents the bandwidth of the transmitted signal. By using the parameters from Table 1 the frequency sweep of the Chirp is calculated as follows

$$k = \left(\frac{B}{T}\right) \tag{3.6}$$

The modeled received signal will be used as an input for two identical matchedfilters. The only difference between both matched-filters is that the input of one of them is modeled using a first order approximation and the input of the second matched-filter will be modeled using a full Doppler compensation. The matched-filter will perform a cross-correlation between the input signal and the transmitted signal. The basic match filter block diagram is shown in Figure 1 and the successive steps to implement the compensation are shown in Figure 2.



Figure 1. An example of matched-filter used in a radar receiver block (Barton [12])

The matched-filter as shown in Figure 1 is implemented as an auto-correlator, wherein the reflected signal is cross-correlated by the transmitted signal. [12].



Figure 2. Block diagram of the process implemented to simulate the outputs of the matched-filter

For an ease in simulation, the target's velocity was assumed to be 0.3% of the speed of light. Another reason for choosing such a high target velocity is to show the fundamental existence of an error when large time-bandwidth product signals are compensated using the first order approximation technique. The outputs of both matched-filters were analyzed to observe the effects as the time-bandwidth product of the transmitted signal was increased from 250 to 5000.

#### **Results of the compensated signals**

By applying the parameters shown on Table 1, the reflected signal was compensated with the first order approximation and full Doppler compensation techniques independently. Both of these signals were used as inputs for two identical matched-filters. The simulated outputs obtained from the matched-filters are shown in Figures 3-7. The first simulation was performed assuming a modest time-bandwidth product of 250 for the transmitted signal. By looking at Figure 3 it can be clearly seen the similarity between both outputs. In this case the difference between the outputs could be neglected.



Figure 3. The output of the matched-filter for a TBW product of 250

The second simulation was performed assuming a time-bandwidth product of 3000 for the transmitted signal. The outputs observed from both matched-filters are shown on Figures 4 and 5. It is clear how the mismatch between both outputs has increased. Finally, the last simulation was performed assuming a time-bandwidth product of 5000. The outputs of both matched-filters are shown in Figures 6 and 7. By looking at

these figures it is clear that the difference between both outputs has increased with respect to the previous results.



Figure 4. The output from the matched-filter used for a TBW product of 3000



Figure 5. Close-up of the matched-filter outputs from figure 3 for a TBW product of 3000



Figure 6. The outputs from the matched-filter for a TBW product of 5000



Figure 7. Close-up of the matched-filter outputs from figure 6 for a TBW product of 5000

However, the question arises: How significant are these differences? From previous results, it is noticed that the time difference ( $\Delta t$ ) between the two outputs in Figures 4 and 6 increases upon increasing the time-bandwidth product. This difference in time could be translated as a difference in range by using the following expression

$$\Delta x = \Delta t \times c \tag{3.7}$$

where  $\Delta t$  corresponds to the difference in time between both outputs and  $\Delta x$  can be interpreted as the difference in range. Figure 8 shows the difference in range from multiple time-bandwidth products. It can be clearly seen that the increment in timebandwidth product is directly proportional to the difference in range. This clearly supports the argument that the errors produced from using the First order approximation does not only depend on the target's velocity, but also depend on the time-bandwidth product of the transmitted signal.



Figure 8. Variation of  $\Delta x$  as a function of the TBW product

#### **Compensation of a radar image**

For the last part of this research, the radar image of a reflected signal is generated using Fourier transformation techniques. For simulation purposes a specific MiG-29 fighter aircraft which is reduced to a point object is considered as the target. It is assumed that the target is moving linearly in the radial direction with a constant velocity v. At the beginning of the simulation, the target is located at 50 m with respect to the radar antenna. In order to modify the time-bandwidth product of the transmitted signal, the bandwidth B is increased exponentially and the duration of the signal is increased linearly. The purpose of this simulation is to observe the differences on the radar image when the first order compensation and the full Doppler compensation are implemented. The initial parameters used in this simulation are shown in Table 2 [13].

Initial Frequency $f_0$	89.73 GHz
Bandwidth	0.375 GHz
Velocity of Target	765.65 m/s (≈Mach 2.25)
Initial TBW Product	756
Final TBW Product	4532

Table 2. Parameters used to simulate the processed radar image of the aircraft

#### **Results of the radar image upon compensation**

In order to simulate the radar image from a moving target, the center frequency  $(f_c)$  of the transmitted signal was set to 80 GHz and the target's cross and slant range resolutions are 1.5m each. By applying the sampling theorem technique, the initial frequency as shown in Table 2 is calculated by subtracting the center frequency with half of the bandwidth. Subsequently the time and frequency vectors of the target are compensated independently by applying these two techniques. Using an approach similar to Figure 2, the image of the target being compensated is generated by applying the FFT algorithm from MATLAB. The compensated images are auto-correlated in the matched filter and the result is shown in Figure 9. From Figure 10 to 13 it can be observed a

considerable deviation of the radar image when the first order approximation is used and the time-bandwidth product of the signal is incremented. Figure 14 is plotted using the results from Table 3, and it provides an insight of the resulting effect of increasing the time-bandwidth Product.

Time Bandwidth	First Order		Full Doppler		Distance between	
Product of the signal	Approximation		Compensation		the points [meters]	
	Х	Y	Х	Y	the points [meters]	
201	63.5	96.5	63.5	95.0	1.5	
403	63.5	77.0	63.5	75.50	1.5	
1194	63.5	39.5	63.5	36.5	3.0	
1911	63.5	20.0	63.5	17.0	3.0	
4532	63.5	81.5	63.5	77.0	4.5	

Table 3. Positions of the maximum peak value of the target aircraft's image and corresponding distances

Table 4. Specifications of the target[14]

Specifications of the Target	Dimensions
Length	17.37 m
Wingspan	11.4 m
Maximum Velocity	2400 km/h

The highlighted column of Table 3 shows the distance error of 4.5 meters produced when the radar image is detected using first order approximation. With reference from Table 4, this corresponding distance gap becomes a significant error for the dimensions listed. Hence for this example, the maximum allowable TBW to detect the target is about 2000.



Figure 9. Target image at time-bandwidth product of 201



Figure 10. Target image at time-bandwidth product of 403



Figure 11. Target image at time-bandwidth product of 1194



Figure 12. Target image at time-bandwidth product of 1911



Figure 13. Target image at time-bandwidth product of 4532



Figure 14. Plot of TBW with the distance between the maxima of the targets (Table3)

# **Chapter Four**

### Conclusion

A comprehensive analysis on how the increment of the time-bandwidth product on the transmitted signal increases the errors generated from using the first order approximation to model the reflected signal was presented. This analysis was performed using two methods. The first method was implemented by modeling the reflected signal using a first order approximation and a full Doppler compensation. These signals were used as inputs for identical matched-filters in which the transmitted signal was used as the reference signal. By analyzing the outputs of both matched-filters, it was observed that the deviation between the output from the matched-filter in which the input was modeled using the full Doppler compensation and the output from the matched-filter with the signal using the first order approximation was increased as the time-bandwidth product of the signal was increased. For this simulation the target velocity was assumed constant and the only parameter that was allowed to be increased was the time-bandwidth product of the transmitted signal. These results show that even if the target's velocity is kept constant, the errors produced by the neglected terms from the First order approximation increases as a function of the time-bandwidth product. Even more, it was shown that a linear dependency exists between the increase of the error produced from using the first order approximation and the time-bandwidth product.

The second method consisted of generating a radar image from the echo signal received from an aircraft reduced as a point target. Upon incrementing the time-bandwidth product, the results obtained from the simulation were in agreement with the first simulation. In other words, the compensation of the signal using a full Doppler compensation gives a better resolution and more accurate position of the target's image.

In conclusion, this shows that a full Doppler compensation should be used to model the reflected signal to prevent any errors produced by the target velocity and time-bandwidth product.

With this work, the analysis on the application of compensation techniques that affect the resolution of the signal reflected from the target is shown. However, additional scope of work is desired by applying this technique to detect multiple targets with MTI Radars. In addition, as future radars continue to have larger time-bandwidth products, a more robust signal processing technique is desired for increased computational efficiency.

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## Appendix A: Matlab code to plot the compensation of the radar signal

```
%Matlab code for simulating the compensation techniques of radar signal
clc
close all
clear all
f1=input('Enter the value of F1 in Hz'); %Frequency
B1=input('Enter the value of BAndwidth in Hz'); % Bandwidth
 TBW=input ('Enter the Time BAndwidth Product'); %Time Bandwidth Product
for m=1:2
T1=m*TBW/(B1);
dt=1/(6*f1);
t=0:dt:T1;
length(t);
dfd=1/T1;
k=B1/T1;
c=3*10^8;
v=3*10^5;
y=exp(j*((2*pi*f1*t)+(pi*k*t.^2))); % Linear FM signal
T=0:dt:2*T1;
length(T);
    al=(c-v)/(c+v); % Full Doppler Compensation
    a2=1-(2*v./c); % First order approximation
    y1=exp(j*((2*pi*f1*a1*t)+(pi*k*(a1*t).^2)));
    y2=exp(j*((2*pi*f1*a2*t)+(pi*k*(a2*t).^2)));
    Y1=abs(xcorr(y,y1)); % Match Filter
    Y2=abs(xcorr(y,y2)); % Match Filter
 figure(m)
  plot(T,Y1,T,Y2,'--');
  grid on;
   title('Matched-filter Output for TBW of 1000');
   xlabel('Convolution Time in seconds');
   ylabel('Amplitude of the signal');
   legend('Full Doppler Compensation','First order approximation');
 end
%To Calculate the distance between the peaks of the two compensated
signals
11=0.5*max(Y1);
modx1=find(Y1<l1);</pre>
iter1=1;
while(1)
    ml=modx1(iter1+1)-modx1(iter1);
```

# Appendix A (Continued)

```
iter1=iter1+1;
    if m1~=1
        iter1;
        break
    end
  end
modx2=find(Y2<11);</pre>
iter2=1;
while(1)
    m2=modx2(iter2+1)-modx1(iter2);
    iter2=iter2+1;
    if m2~=1
        iter2;
        break
    end
 end
figure(1)
T(iter1);
T(iter2);
w1=T(iter1)-T(iter2)
Dist=(T(iter1)-T(iter2))*c
  plot(m*TBW,Dist,'*');
  grid on
 hold on
 end
 title('Relationship between TBW and Distance Gap');
 xlabel('Time Bandwidth Product');
 ylabel('Distance Gap in m');
```

Appendix B: Matlab code to plot the compensation of a radar image

```
for i=1:n
    1=0;
    for k=1:m
        t(i,k)=jj*T2;
                       %creating time matrix
        f(i,k)=f0+l*df; %Creating frequency matrix
        jj=jj+1;
        1=1+1;
    end
end
R1 = (R + (v * t));
%Compensation of the Signal
H=exp(li*4*pi*f.*R1/c);
 H1=exp(li*4*pi*a2*f.*R1/c).*exp(-(li*2*pi*a1*f.*v*t));
 H2=exp(li*4*pi*a2*f.*R1/c).*exp(-(li*2*pi*a2*f.*v*t));
 hh=H1;
 %Generating the image of the target
  for i=1:n
     H1(i,:)=(fft(H1(i,:)));
     H2(i,:) = (fft(H2(i,:)));
 end
for k=1:m
    H1(:,k) = abs(fftshift(fft(H1(:,k))));
    H2(:,k) = abs(fftshift(fft(H2(:,k))));
end
%Normalizing and converting to dB scale
for k=1:m
    Max1(k) = max(H1(:,k));
    Max2(k) = max(H2(:,k));
end
Max 1=max(Max1)
Max<sup>2</sup>=max(Max2)
for i=1:n
    for k=1:m
        H norm1(i,k)=H1(i,k)/Max 1;
        H norm2(i,k)=H2(i,k)/Max 2;
        H db1(i,k)=20*log10(H norm1(i,k));
        H db2(i,k)=20*log10(H norm2(i,k));
        if H db1(i,k)<-20
            H db1(i,k)=-20;
        end
        if H db2(i,k)<-20
             H db2(i,k) = -20;
        end
```

### **Appendix B** (Continued)

```
end
end
% calculating the peak points of respective compensation technique
[x indx1,y indx1]=find(H norm1==max(max(H norm1)))
K1(p) = (x indx1*rs) -1;
L1(p) = (y_indx1*rc) - 1;
[x indx2, y indx2]=find(H norm2==max(max(H norm2)))
K2(p) = (x indx2*rs) - 1;
L2(p) = (y indx2*rc) -1;
%Plots
N=0:(n-1);
M=N;
figure(p)
subplot(2,1,1)
imagesc((N.*rs), (M.*rc), -(H norm1))
xlabel('Amplitude rs');
ylabel('Amplitude rc');
colormap(gray)
colorbar
title('First order approximation');
subplot(2,1,2)
imagesc((N.*rs), (M.*rc), -(H norm2))
xlabel('Amplitude rs');
ylabel('Amplitude rc');
colormap(gray)
colorbar
title('Full Doppler Compensation');
end
 for g=1:5
    dL(g) = (L1(g) - L2(g));
    dK(q) = (K1(q) - K2(q));
    Dist1(g) = sqrt(((dL(g))^2) + ((dK(g))^2));
 end
figure(p+1)
subplot(2,1,1)
plot(TB,dL,'*',TB,dK,'*')
xlabel('Time Bandwidth Product of the reflected signal');
ylabel('Difference of distance between targets ');
title('Plot of varying difference in distance of compensation technique
Vs Time Bandwidth Product');
legend('Slant distance','Cross distance');
subplot(2,1,2)
```

# Appendix B (Continued)

plot(TB,Dist1)
xlabel('Time Bandwidth Product of the reflected signal');
ylabel('distance between targets ');
title('Plot of varying distances for individual compensation technique
Vs Time Bandwidth Product');