A Compressive Radar System with Chaos Based FM Signals Generated Using the Bernoulli Map

Charan Teja Enugula

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A COMPRESSIVE RADAR SYSTEM WITH CHAOS BASED FM SIGNALS GENERATED USING THE BERNOULLI MAP

by

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A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Electrical Engineering
Department of Electrical Engineering

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The University of Texas at Tyler
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Abstract

A COMPRESSIVE RADAR SYSTEM WITH CHAOS BASED FM SIGNALS GENERATED USING THE BERNOULLI MAP

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The University of Texas at Tyler
May 2012

Matched filters are used in radar systems to identify echo signals embedded in noise. By using matched filters, range and Doppler information about the target can be extracted from the reflected signal. The use of matched-filters in high frequency radars has the effect of increasing the cost and complexity of these systems. For that reason, the radar research community is looking at a new technique called compressive sensing or compressive sampling. This technique has the potential to eliminate the use of matched filters and high frequency analog-to-digital converters. Compressive sensing works by exploiting the signal sparsity. Typically, in compressive sensing the received signal is sampled below the Nyquist rate. This is possible by taking a small number of random projections of the signal, which contain most of the important information. In the past, compressive sensing has been successfully applied in the fields of image processing, medical sciences and wireless sensors. In this research work, it is proposed that compressive sensing be applied to a chaotic radar system. The goal is to eliminate the need of a matched filter and at the same time increase the target resolution.

In the first stage of this research work, a chaos based FM signal was generated using a Bernoulli map sequence. In a typical radar system, the received signal must
be sampled according to the Nyquist rate, which corresponds to at least twice the maximum frequency of the signal. However, in compressive sensing the received signal is sampled at a lower rate and employs numerical algorithms to reconstruct the radar scene. In order to model the reflected signal, a chaos based FM signal is rearranged into a matrix and multiplied by the vectorized radar scene. The attained over sampled set of equations cannot be solved using compressive sensing. For that reason, the reflected signal is under-sampled by removing rows from the received signal. This is equivalent to sampling the received signal at a slower rate. The scene is then reconstructed using a minimal set of observations of the transmitted signal. The undetermined set of equations is solved using an optimization technique called “convex programming” or cvx. The plots of the original vectorized scene are compared against the reconstructed vectorized scene in order to verify the reliability of this method.

The simulation results show that the compressive sensing was capable of recovering the radar scene when stationary targets and non-stationary targets were considered. It was also observed that even when the matched filter was capable of recovering the radar scene there was a considerable amount of noise introduced by the matched filter, making it difficult to recognize and identify the targets.
Chapter One

Introduction

Understanding the basic functionality of radar is simple, whereas its actual implementation is more complex. The radar operates by transmitting electromagnetic waves, when these waves collide with a metallic object (target) the signal travels back (echo) to the transceiver. The properties of the received signal provide enough information about the target so that it is possible to identify it from the other objects. Typically, the received signal is compared against a reference signal using a matched filter. The output of the matched filter provides information about the range and the angular location of the object. The target’s range is calculated using the time taken by the signal to travel from the radar to the target and back to the radar. The angular location of a moving target is calculated using the frequency shift induced in the received signal, which is typically called the “Doppler shift”.

Most of the advances in radar systems are aimed to improve their resolution. As a result, their operating frequency has been increased from 10GHz up to 94GHz, and new millimeter-wave (100-300GHz) radar systems are currently being proposed. One of the major concerns with these frequencies is the large bandwidth required by the matched filter and the analog-to-digital converters. Both of these components require the signal to be sampled at the Nyquist rate. Therefore, a huge burden is placed on these components that need to work at very high frequencies, and process massive amounts of data.

Compressive Sensing (also known as Compressive Sampling) is proposed as a solution to overcome the problems mentioned before. A requirement for compressive sensing is that the signal needs to be sparse in nature. Also, using this technique a signal can be reconstructed even if it is sampled below the Nyquist rate. This method also provides a complete new way to reconstruct the signal using optimization tech-
techniques and a minimum number of observations. When compressive sensing is applied, only the important information within the signal is acquired rather than acquiring information that will be eventually discarded at the receiver. This method employs non-adaptive linear projections to preserve the structure of the signal. One of the major advantages of compressive sensing is that, by using optimization techniques a perfect reconstruction of the signal is possible if the proper parameters have been selected. By implementing this technique, a new approach for radars is proposed. Compressive sensing implements optimization techniques like $l_1$ minimization, the convex optimization and other basis pursuits to reconstruct the original scene from a minimum number of samples of the received signal. In this thesis, compressive sensing is used to eliminate the need for the matched filter and high speed analog-to-digital converters. This will result in a cost reduction of the radar hardware and an optimum utilization of the bandwidth.

Besides Compressive sensing, the choice of the radar signal directly affects the performance of a range-Doppler imaging system. The use of chaotic signals in radar systems has advantages as they behave like pseudo noise, they have a wide band, and they are easy to generate [1]. On the other hand, a frequency modulated (FM) signal provides high resolution, high transmitting power, low design cost and low probability of intercept and interference. The chaos based frequency modulated (CBFM) signal is generated using the Bernoulli chaotic map. The spectrum of the FM signal can be modified by changing the probability density function (pdf) of the chaotic sequence. The chaotic sequence is pseudo random in nature and has properties similar to a truly random signal. Because of that the CBFM signal is a popular choice in radar systems.

The compressive sensing technique involves the creation of what is called a measurement matrix, and it is created using the CBFM signal. The measurement matrix is then multiplied by the scene vector to generate the received signal. The radar scene is recovered using compressive sensing, this is performed by under-sampling the received signal and by using optimization techniques.

In summary, according to the Shannon-Nyquist sampling theorem, a signal should be sampled at least at twice the maximum frequency. However, the compressive sensing technique allows reconstructing the radar scene by sampling the received signal below the Nyquist rate. As radar signals are sparse in nature, the use of compressive sensing yields a perfect reconstruction of the radar scene. Using MATLAB a
CBFM signal is generated using a Bernoulli chaotic map. This signal was used to generate the measurement matrix. This matrix was multiplied by the scene vector, which consists of multiple targets. The received signal contains information about the target location. Finally, the scene vector was reconstructed using the under sampled received signal, the measurement matrix and a convex optimization technique.

1.1 Organization of Thesis

This thesis is divided into five chapters. Chapter 2 discusses compressive sensing and the optimization techniques used to recover the radar scene. It also provides an overview of radar systems and compressive radar. Chapter 3 describes the implementation of compressive sensing on the radar system and Chapter 4 presents the reconstructed scene from the simulations. Chapter 5 includes the conclusions and future work.
Chapter Two

Background

2.1 Introduction to Compressive Sensing

Compressive sensing, also known as compressive sampling (CS), emerged as a theory around the year 2000, and exploded in the following years. It is a new approach where random samples are acquired by a simple matrix vector multiplication. For example if compressive sensing is applied to an image, the number of samples (number of rows in the matrix) obtained from this multiplication is less than the number of pixels in the image (number of columns in the matrix). The image can then be reconstructed with the reduced number of samples using an optimization technique. One recent application for CS is the encryption of data. Encryption using CS is based on the idea that the sampling matrix used to compress the image also encrypts the data by mapping it into a new set of basis functions [2]. Without knowledge of either the sampling matrix, or the key used to create the sampling matrix, one cannot decrypt the original data which is transmitted.

Compressive sensing allows reconstruction of a signal that has been sampled below the Nyquist rate. Conventional sampling theory establishes that digital samples of an analog signal must be sampled at a sufficient rate for the signal to be reconstructed without aliasing; this is commonly known as the Nyquist sampling frequency [3]. This type of sampling reconstructs the signal accurately and requires bandwidth of at least twice the maximum frequency of the signal to be sampled. This technique strongly depends on exploiting the signal’s sparsity. Sparsity can be defined as, the combination of a small number of projections on a particular basis, which implies that the same signal can be represented with a smaller amount of data with a high possibility of accurate reconstruction [4].
Typically in non-compressed sensing methods, the first step is to acquire large amounts of data, then project them into an appropriate basis, and transmit the projections [4]. This is a waste of resources, since many data points are initially collected and transmitted. In compressed sensing, it is necessary to find a basis that will approximately represent any sparse signal. As there are only few samples involved in this process the sampling time is short. Compressive sensing employs non-adaptive linear projections to preserve the structure of the signal [5]. The following sections describe more in detail how compressive sensing works and is applied.

2.1.1 Signal Representation and Measurement Matrix

Signal representation plays a vital role in compressive sensing. Consider a real-valued, finite length, one dimensional, discrete time signal $x$, which can be viewed as a $N \times 1$ column vector in $R^N$ with elements $x[n]$, $n = 1, 2, \ldots, N$. Any signal in $R^N$ can be represented in terms of a basis of $N \times 1$ vectors $\{\Psi\}_{i=1}^{N}$ [6]. If the basis is an orthonormal matrix $\Psi = [\psi_1 | \psi_2 | \ldots | \psi_N]$ with the vectors $\{\psi_i\}$ as columns, then the signal $x$ can be expressed as

$$x = \sum_{i=1}^{N} s_i \psi_i \quad \text{or} \quad x = \psi s \quad (2.1)$$

where $s$ is a $N \times 1$ column vector. Furthermore, $x$ is an equivalent representation of the signal in the time-domain and $s$ is an equivalent representation of the signal in the $\Psi$ domain. Even more, the signal $x$ is $K$-sparse, which means that only $K$ samples of the signal have non-zero values. The signal $x$ is compressible if it has a few large coefficients and many small coefficients, and $K \ll N$, where $N$ is the length of the signal.

Compressive sensing addresses the inefficiencies of transform coding by acquiring a compressed signal directly without going through the intermediate stage of acquiring $N$ samples [7, 8]. In terms of signal acquisition, CS measures a signal $M$ times which is approximately equal to $K$, instead of the total length of the signal. This is possible by making $N$ non-adaptive linear observations in the form of $y = \Phi x$, where $\Phi$ is a dictionary of size $M \times N$. Substituting equation 2.1 in $y$, then the equation for the linear observations can be written as

$$y = \Phi x = \Phi \psi s = \Theta s \quad (2.2)$$
where $\Theta = \Phi \Psi$ is an $M \times N$ matrix, $\Phi$ is the uniform random measurement matrix which is fixed and does not depend on signal $x$, and $y$ is the measurement vector of the signal $x$.

\[
\begin{align*}
\Phi & \quad \Psi \\
M \times N & \quad x = \Psi \alpha
\end{align*}
\]

Figure 2.1: Compressive sensing measurement matrix [9]

Designing a stable measurement matrix is a very important factor in compressive sensing. There are two types of measurement matrices that can be used: the random measurement matrix and predefined measurement matrix. The measurement matrix $\Phi$ shown in figure 2.1 must allow the reconstruction of the length-$N$ signal $x$ from $M < N$ measurements. As the number of measurements made is less than the length of the signal, reconstruction by traditional methods is not possible. However, as mentioned before if $x$ is $K$-sparse and the $K$ locations are known, then the problem can be solved provided that $M \geq K$ [6]. The following condition should be satisfied for any vector $v$ sharing the same $K$ nonzero entries as the vector $s$

\[
1 - \epsilon \leq \frac{\| \Theta v \|_2}{\| v \|_2} \leq 1 + \epsilon \tag{2.3}
\]

where $\epsilon$ is going to be greater than zero ($\epsilon > 0$). A sufficient condition for a stable solution for both $K$-sparse and compressible signals is that $\Theta$ should satisfy equation 2.3 for an arbitrary $3K$-sparse vector $v$. This condition is referred to as the restricted isometry property (RIP) [7]. If the rows from measurement matrix $\Phi \phi_j$ cannot sparsely represent the columns $\psi_i$ of $\Psi$ and vice versa it is referred to as incoherence. If $\Phi$ is sufficiently “incoherent” and satisfies the “Restricted Isometric Property (RIP)”, then the information of $s$ will be implanted in $y$ such that it can be perfectly recovered with high probability. Both the RIP and incoherence can be achieved with high probability simply by selecting the measurement matrix as a random matrix. As
mentioned above if a signal $x$ composed of $N$ samples is sparse then the actual signal can be perfectly reconstructed using $M \geq cK \log(N/K) \ll N$ linear projection of $x$ onto another basis.

### 2.2 Reconstruction Algorithm

In compressive sensing optimization techniques are used to recover the original signal from a minimal number of observations. These optimization techniques include: the $l_1$ minimization, the convex optimization and other greedy algorithms such as the *orthogonal matching pursuit* (OMP) [10]. These techniques play a vital role in recovering the signal with high probability. The signal reconstruction algorithm must take the $M$ measurements in the observation vector $y$, the measurement matrix $\Phi$, and the basis $\Psi$ in order to reconstruct the signal $x$ composed by $N$ samples. For example, any $K$-sparse signal in which $M < N$ in equation (2) there are infinite number of solutions that satisfy the condition $\Theta s^t = y$.

#### 2.2.1 $l_p$ Norm Reconstruction

The $l_p$ norm reconstruction algorithm is used to reconstruct a sparse signal $x$. A potential approach to the sparse signal problem is minimizing the $l_p$ norm function of $x$. For maximum computational efficiency the $l_p$ norm reconstruction is classified as Min-$l_2$, Min-$l_0$ and Min-$l_1$. First define the $l_p$ norm of the vector $s$ as

$$
(\| s \|_p)^p = \sum_{i=1}^{N} | s_i |^p
$$

(2.4)

The typical approach for these type of inverse problems is to find the vector in the translated null space with the smallest $l_p$ norm by solving the following expression

$$
\hat{S} = \text{argmin} \| s^t \|_2 \text{ such that } \Theta s^t = y
$$

(2.5)

This optimization has the convenient closed-form solution

$$
\hat{S} = \Theta^T(\Theta \Theta^T)^{-1} y
$$

(2.6)

However, the $l_2$ solution is not a $K$-sparse solution as it returns many non-zero elements for $\hat{S}$. 

7
Unlike the $l_2$ norm, the $l_0$ norm can recover a $K$-sparse signal exactly with high probability using the modified optimization

$$\hat{S} = \text{argmin} \| st \|_0 \text{ such that } \Theta st = y$$

(2.7)

The problem with the $l_0$ norm is solving equation 2.7 is numerically unstable [6].

The $l_1$ norm optimization is capable of perfect recovery of $K$-sparse signals and closely approximates the compressible signals with high probability using measurements given by $M \geq cK \log(N/K)$ [7, 8].

$$\hat{S} = \text{argmin} \| st \|_1 \text{ such that } \Theta st = y$$

(2.8)

2.2.2 Convex Programming (cvx)

Disciplined convex programs or DCP’s, developed by Michael Grant, Stephen Boyd, and Yinyu Ye are convex optimization problems which are described using a set of construction rules, which enables the problems to be analyzed and solved efficiently.[11]. cvx is a modeling system for disciplined convex programming. The main advantage is that it can greatly simplify the task of specifying and solving the problem compared to the other optimization techniques such as linear programs (LP’s), quadratic programs (QP’s), second-order cone programs(SOCP’s) and semi-definite programs (SDP’s) [11]. It can also be used to solve other problems which involve non-differentiable functions, such as the $l_1$ norm [11]. Problems which disobey the DCP rule set are strictly rejected even if the problem is convex. The cvx programming is described with the following example from cvx user guide [11].

Consider the most basic convex optimization problem which is the least-squares. In this problem to find $x \in \mathbb{R}^n$ that minimizes $\| Ax - b \|_2$, where $A \in \mathbb{R}^{m \times n}$ in which $m \geq n$ and $\text{Rank}(A) = n$. For example, if $m$, $n$, $A$ and $b$ are given some values, the problem is solved in MATLAB then the least square solution $x = (A^T A)^{-1} A^T b$ is easily computed using $x = A \backslash b$. Using cvx the problem is solved as follows

```matlab
cvx_begin
    variable x(n);
    minimize(norm(A*x-b));
cvx_end
```
The first line of code creates a placeholder for the new cvx specification and prepares MATLAB to accept variable declarations, constraints and an objective function. The second line declares the optimization variable $x$ of size $n$. The third line specifies an objective function to be minimized such as $l_2$ norm of $Ax - b$ for the above case. The last line tells the end of cvx and the problem to be solved.

### 2.3 Compressive Sensing Applications

#### 2.3.1 Compressive Imaging

The common approach in digital imaging systems is to capture as many pixels as possible and later compress the captured image by digital means. Image compression algorithms can reduce data sets by order of magnitude (for example pixels) which makes the systems capturing high-resolution images feasible. Image compression algorithms convert high-resolution images into a relatively small bit streams by keeping the essential features intact. But the compressive sampling does not compress the total large data set into a small one but it transforms the image into an appropriate basis and then codes only the important coefficients of the total data [12].

#### 2.3.2 Medical Imaging

Magnetic Resonance Imaging (MRI) is an essential medical imaging tool burdened by an inherently slow data acquisition process [13]. The application of compressive sensing to MRI has the potential for significant scan time reductions, with benefits for patients and health care economies. MRI obeys two important requirements for successful applications of compressive sensing (1) medical imagery is compressible by sparse coding in transform domain (2) MRI scanners acquire samples of encoded image in spatial frequency, rather than direct pixel samples [13]. The CS technique can be applied to improve the MRI scan time and the image resolution.

#### 2.3.3 Analog to Information Converter

Analog to-digital converters (A/D) have been used in sensing and communications due to the advancements in digital signal processing. A/D converters work on the principle of Nyquist theorem where the analog data needs to be sampled at a rate of at least twice the highest frequency of the signal being processed. In many of
the applications that require compressing, the data is compressed after the signal has been sampled at Nyquist rate. As proposed by Baraniuk, recent developments in compressive sensing will help in the design of a smart sampling technique called Analog to Information converter (A/I) to acquire only the important information [14].

2.3.4 Compressive Radar

In compressive radar the main goal is to eliminate the need for the matched-filter and the analog to-digital converter which in turn reduces the complexity and the cost of the receiver hardware. As a result, a received radar signal can be reconstructed with fewer measurements by solving an inverse problem through some optimization technique [15]. This research work discusses the implementation of compressive sensing in chaotic radar, which typically depends on the use of a matched filter to detect the target.

2.4 Radar

Radar is the acronym for Radio Detection and Ranging. The basic radar works by obtaining information about the target’s range and the target’s angular velocity. The range is obtained by measuring the echo generated when the remote object is illuminated with the transmitted radar signal. On the other hand, the angular position of the target is determined by the Doppler shift in the received signal. Typically, pulse radars are used to measure the target range by transmitting a very short microwave pulse and measuring the delay between transmission and reception [16]. The type of the signal transmitted plays an important role in determining the range and the velocity of the target. The extraction of the information from the received signal needs a complex receiving system which is designated as “Matched Filter”.

2.4.1 Matched Filter

The most important characteristic of the matched filter is that it produces the maximum achievable instantaneous SNR at its output when a signal corrupted by noise is present [17]. The matched filter is used in radars to maximize the output peak-SNR and correlates to the spectrum of the signal expected at a particular Doppler shift \( f_d \). For this reason, the matched filter is an important component of modern radar
receivers.

The characteristics of a matched filter can be designated by either a frequency response function or a time response function. The Fourier transform is used to relate both functions. In the frequency domain the matched-filter transfer function, $H(\omega)$, is the complex conjugate function of the spectrum of the signal that is to be processed in an optimum fashion [18]. Thus, the equation of the transfer function of a matched filter with an input signal $s(t)$ with spectrum $S(\omega)$ is defined as

$$H(\omega) = kS^*(\omega)\exp[-j\omega T_d] \quad (2.9)$$

and the time response is given by

$$h(t) = ks^*(T_d - t) \quad (2.10)$$

where $T_d$ is a delay constant required to make the filter physically realizable and $k$ is a normalizing factor.

The matched filter processing of an echo signal is the coherent summation of the reflected signal from the target’s reflection points which are spread over the target’s range. In the matched-filter, a match is made for the transmitted signal, irrespective of the target type. A type of matched filter used in high resolution radars is the pulse-compression filter.

### 2.4.2 Signal to Noise Ratio (SNR) and Time-bandwidth Product

The SNR is a measure of the signal strength relative to the background noise. The peak SNR at the output of a radar receiver is maximized if the receiving system is matched to the received signal [19]. Wehner [19] and North [20], state that regardless of the radar waveform, the ratio of peak instantaneous signal to average noise power of the output response of a matched-filter receiver is equal to twice the received signal energy $E$ divided by the noise power per hertz, $N_0$.

$$\left(\frac{\hat{S}}{N}\right)_{out} = \frac{2E}{N_0} \quad (2.11)$$

where $N_0$ is defined for a one-sided spectrum of frequencies. From the previous equation the SNR over pulse duration $T_1$ is given by
\[
\left( \frac{\hat{S}}{N} \right)_{\text{out}} = \frac{2ST_1}{N/\beta_n}
\]  \hspace{1cm} (2.12)

where \( N \) is the available input noise power within the matched-filter receiver noise bandwidth \( \beta_n \). The input SNR in terms of peak output SNR is expressed as

\[
\left( \frac{\hat{S}}{N} \right)_{\text{in}} = \frac{1}{2T_1\beta_n} \left( \frac{\hat{S}}{N} \right)_{\text{out}}
\]  \hspace{1cm} (2.13)

The quantity \( T_1\beta_n \) in equation 2.13 is the time-bandwidth product of the transmitted signal. The factor by which the SNR at output is increased over the input (signal processing gain) approaches the time-bandwidth product \( T_1\beta_n \) for ideal matching. Phase or frequency coding techniques are used to make the time-bandwidth product greater than unity.

### 2.4.3 Ambiguity Function

The ambiguity function is the most complete statement in revealing a radar waveform’s inherent performance. The ability to resolve a target in range and velocity can be assessed by directly examining the ambiguity function \( \chi(\tau, \nu) \) or the ambiguity surface \( |\chi(\tau, \nu)|^2 \) \cite{21}, \cite{22}. The ambiguity function is defined as

\[
\chi(\tau, \nu) = \int s(t)s^*(t-\tau)\exp(j2\pi\nu t)dt
\]  \hspace{1cm} (2.14)

where \( \tau \) is the time delay, \( \nu \) is the Doppler frequency and \( s^*(t) \) is the complex conjugate of the transmitted signal. The ambiguity function can also be called a time-frequency correlation function of the signal \cite{23}.

The shape of the ambiguity surface is entirely dependent upon the waveform parameters. The definition of the area under the curve of the ambiguity function is always one. A normalized expression is obtained by

\[
\int_{-\infty}^{\infty} |s(t)|^2 dt = 1
\]  \hspace{1cm} (2.15)

The peak of \( |\chi|^2 \) occurs at \((\tau = 0, \nu = 0)\) and has an intensity equal to the energy \( E \) of the process. Typically the shape of \( |\chi|^2 \) looks like a bed of nails, a ridge or a thumbtack. The shapes of the ambiguity surfaces are chosen according to design preferences.
2.5 Compressive Radar

The implementation of compressive sensing on radars is an emerging concept which can be illustrated by the following example. Consider $K$ targets with un-known range-velocities and corresponding reflection coefficients. The time-frequency plane is divided into an $N \times N$ matrix, in which each point represents a unique time-frequency shift $H_i$. The number of targets is much smaller than the number of points in the matrix $K \ll N^2$. This implies that the time-frequency plane is sparse. This is one of the most important conditions in order to apply compressive sensing. By vectorizing the scene matrix, it can be represented as a $N^2 \times 1$ sparse vector $s$.

In order to apply compressive sensing to the radar scene, assume that a pseudo random signal is transmitted. The received signal now is of the form

$$y = \sum_{i=0}^{N^2-1} s_i H_i f = \Phi s$$

and if the number of targets obey the sparsity constraints, then the original target can be recovered using compressed sensing techniques [10]. The resolution of the recovered target scene depends on the discretization of the time-frequency plane as it has $N^2$ unique time-frequency shifts. That is, multiple targets located at adjacent points in the time-frequency plane can be recovered due to the nature of the compressed sensing reconstruction technique. Three main characteristics are noted here: (1) the transmitted signal must be incoherent, (2) there is no matched-filter and (3) compressed sensing techniques are used to recover the sparse targeted scene instead.

The main advantage of compressive sensing over the matched filter is the elimination of side lobes of the received signal, which eliminates the problem of identifying targets that are close together. In CS the location of an object is observed as a spike at a particular point rather than a signal with a main lobe and multiple side lobes. The use of compressive sensing also eliminates the need for a fast A/D converter, which in turn reduces the receiver A/D conversion bandwidth. The elimination of the matched filter and the A/D converter in a radar system implies an overall reduction in the cost of these systems.
Chapter Three

Analysis and Design

3.1 Introduction

In this chapter, a technique to improve the resolution in radar systems using compressive sensing is proposed. The goal is to eliminate the need for the matched filter, which in turn reduces the cost of the system. Even more, in the proposed system the received radar signal is sampled below the standard Nyquist rate. The under sampled signal is transmitted over the network and successfully reconstructed at the receiver using compressive sensing without losing any important information. The following subsections discuss the implementation of compressive sensing for two different cases. The first scenario treats the reconstruction of a radar scene for stationary targets, and the second scenario treats the reconstruction of a radar scene for moving targets.

3.2 Compressive Sensing on Stationary Targets

In order to study how to reconstruct a radar scene when stationary targets are considered, a chaos based FM signal generated using a Bernoulli map sequence is used as the transmitted signal. A time domain plot of the signal is shown in Figure 3.1. The expression used to generate the chaos based FM signal is given by equation 3.1.

\[ s(n) = \text{Re}(A \exp(j2\pi Kx(n))) \]  

where \( A \) is the amplitude of the signal, \( K \) is the modulation index, and \( x(n) \) is a discrete version of a chaotic function. It was decided to use the Bernoulli map sequence to generate the chaos based FM signal, because it exhibits line trajectories in the pseudo-phase space. It was also determined that the Bernoulli random frequency modulation (RFM) was unique, because of its highly decorrelated samples, which
yields a white spectrum [23]. The expression used to generate the Bernoulli sequence is given by equation 3.2

\[ \varphi(n+1) = \begin{cases} B\varphi(n) + A & \text{if } \varphi(n) < 0 \\ B\varphi(n) - A & \text{if } \varphi(n) > 0 \end{cases} \] (3.2)

where the parameters used to generate this sequence are:

\[ B = 1.7 \]
\[ \varphi_0 = \epsilon [-1/2, 1/2] \]

Once the Bernoulli sequence was generated, the discrete version of the chaotic function is obtained using the following expression

\[ f(n) = \sum_{k=1}^{n} \varphi(k) \] (3.3)

The ambiguity surface of a Bernoulli RFM signal possesses a uniform instantaneous frequency distribution with tail-shifted trajectories, which yield side lobes whose distribution and intensity on the range-Doppler plane are comparable to the ambiguity pedestal of the Gaussian FM [23].

The received signal is generated by performing the convolution between the transmitted signal and the target. In order to perform the convolution using MATLAB and matrix multiplications, the chaos based FM signal is re-arranged into a matrix. In compressive sensing this matrix is called the “measurement matrix”. This matrix is generated by creating time-shifted versions of the transmitted signal in each column of the matrix. An image of this measurement matrix is shown in Figure 3.2. According to Emmanuel Candés and Terence Tao [24] the measurement matrix should satisfy the Restricted Isometric Property in order to be used in compressive sensing.

Once the measurement matrix is created, the next step is to create the radar scene. The radar scene for stationary target is a vector in which each position in the vector represents the position of the target in space. This vector is created by randomly placing objects with different reflectivities. A plot of the radar scene used for stationary targets is shown in figure 3.3, where the horizontal axis represents the distance from the radar to the target, and the vertical axis represents the target’s reflectivity.
The next step is to apply compressive sensing to the received signal in order to recover the radar scene. As mentioned before, the received signal was generated by performing the convolution between the radar scene and the transmitted signal. However, in order to apply compressive sensing the number of samples from the signal should be smaller than the number of samples required by Nyquist. An image of the measurement matrix after under-sampling the received signal is shown in figure 3.4. For compressive sensing it is also necessary that the columns from the measurement matrix be orthogonal. A plot of the correlation between columns from the under-sampled measurement matrix is shown in figure 3.5.

Using the under-sampled version of the received signal and the measurement matrix it is possible to reconstruct the radar scene perfectly. The radar scene is reconstructed using the convex programming optimization technique cvx mentioned in section 2.2.2. This technique will transform the problem into a linear program (LP), and will produce a vector which minimizes the $l_2$ norm of the objective function. Finally, the reconstructed vectorized scene is compared against the original vectorized scene.

![Chaotic behaviour of Bernoulli map](image)

Figure 3.1: Chaotic behaviour of Bernoulli map
3.3 Compressive Sensing on Moving Targets

Compressive Sensing on moving targets is similar to that of stationary targets. The main difference between the two of them is that the scene used for moving targets is a two-dimensional matrix. Where the horizontal axis represents the target’s range,
Figure 3.4: Measurement matrix after under-sampling

Figure 3.5: Autocorrelation of the columns of the measurement matrix

and the vertical axis represents the target’s Doppler. The Doppler shift of the target can be either negative or positive. If the Doppler is negative that indicates that the target is moving away from the radar, if the Doppler is positive, the target is approaching the radar, and zero Doppler is located at the middle of the scene matrix. The velocity and range of the target are determined in real time as the target is
moving. The chaos based FM signal is generated using a Bernoulli map sequence and is used as the transmitted signal. The expression used to generate the transmitted signal is represented by equation 3.4.

$$s(n) = Re(A \exp(j2\pi K (x(n) + f_d)))$$  \hspace{1cm} (3.4)

where $A$ is the amplitude of the signal, $K$ is the modulation index, $x(n)$ is the discrete version of a chaotic function, and $f_d$ is the Doppler frequency. The parameters used to generate the Bernoulli sequence are the same as the ones used for the stationary target. As mentioned before, the generated chaos based FM (CBFM) signal is re-arranged into a matrix, called the measurement matrix. For non-stationary targets, the size of the measurement matrix is larger, because the Doppler shift induced by the target needs to be taken into consideration. Also, the shifting pattern for this matrix is going to be different, as the Doppler frequency will vary. The matrix is generated by creating time shifted versions of the transmitted signal for all the ranges and for each Doppler frequency. Every time the Doppler frequency is changed the time shifting of the signal is reset. An image of the measurement matrix is in the figures 3.6 and 3.7.

The next step is to create a two-dimensional scene matrix composed of multiple targets placed randomly. An image of the scene matrix is shown in figure 3.8. In order to apply compressive sensing, the scene matrix is vectorized. This vector represents all the possible ranges repeating for each Doppler frequency. A graphical representation of this vectorized matrix is shown in figure 3.9.

The received signal is generated by multiplying the scene vector with the measurement matrix. As it was made for the stationary targets, the measurement matrix and the received signal need to be under sampled in order to apply compressive sensing. Also, the measurement matrix should satisfy the RIP. The under sampled version of the measurement matrix is shown in figures 3.10 and 3.11.

At the receiver, the radar scene is reconstructed using a minimal number of observations using the cvx optimization technique. Finally, the reconstructed scene is compared against the original scene.
Figure 3.6: Measurement matrix shifting pattern

Figure 3.7: Zoomed in measurement matrix shifting pattern
Figure 3.8: Scene with multiple targets

Figure 3.9: Vectorized scene matrix
Figure 3.10: Measurement matrix after under-sampling for moving targets

Figure 3.11: Zoomed in reduced set of measurement matrix shifting pattern
Chapter Four

Results

The main goal of this research work is to eliminate the need for a matched filter and reconstruct the scene using optimization techniques. As mentioned in chapter 3, the design and procedures used for the stationary targets, and the moving targets are similar. The main difference between these two is how the scene matrix is created. For stationary targets, the scene only contains information about the target’s range. On the other hand, for moving targets the radar scene provides information about the range and velocity of the target.

4.1 Stationary Targets

For stationary targets, a chaos based FM signal was generated using a Bernoulli map sequence. The parameters used to generate the chaotic signal for the simulation are shown in table 4.1. In order to perform the convolution using matrix multiplications, the chaos based FM signal was re-arranged into a matrix which is called the measurement matrix. This matrix is generated by creating time-shifted versions of the transmitted signal in each column. The radar scene is created by placing multiple objects randomly. For the stationary targets the radar scene is made by a vector of zeros except for the positions where the targets are located. For moving targets the radar scene is a two dimensional matrix in which one direction of the matrix represents the Doppler shift induced by the target’s velocity, and the other direction represents the target’s range.

For the stationary targets, the radar scene was recovered using the convex optimization technique cvx from section 2.2.2 as shown in figure 4.1. The recovered radar scene is compared against the original scene in order to observe the difference between the two. By looking at figure 4.2 it can be clearly seen that by using compressive
sensing the radar scene was perfectly recovered from a limited number of samples. Even more, figure 4.3 shows the recovered radar scene when the matched filter is used. It can be seen that even when the matched filter was able to recover it, there is a lot of noise introduced by the matched filter. Also, it is important to notice that the matched filter requires a full set of samples from the reflected signal.

Table 4.1: Parameters used to generate chaos based FM signal for stationary targets

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Center Frequency</td>
<td>100 MHz</td>
</tr>
<tr>
<td>Sampling Time</td>
<td>1 ns</td>
</tr>
<tr>
<td>Duration of the Signal</td>
<td>1 µsec</td>
</tr>
<tr>
<td>Chaotic Regime</td>
<td>1.7</td>
</tr>
</tbody>
</table>

Figure 4.1: Reconstructed radar scene using cvx

4.2 Moving Targets

For the moving targets a chaos based FM signal was generated using a Bernoulli map sequence. The parameters used to generate this signal are shown in the table 4.2. Similar to the stationary targets, once the chaotic signal was generated; it was re-arranged into a matrix called the measurement matrix. The measurement matrix
is generated by creating time shifted versions of the transmitted signal for all the ranges and for each Doppler frequency. Every time the Doppler frequency is changed the time shifting of the signal is reset.

The radar scene is represented by placing multiple targets randomly. In order to
apply compressive sensing, the received signal and the measurement matrix are under sampled as it was done for the stationary targets.

For the non-stationary targets, the radar scene was recovered using the convex optimization technique cvx from section 2.2.2. The recovered radar scene is compared against the original scene in order to observe the differences between the two. By looking at figure 4.4 it can be clearly seen that by using compressive sensing the radar scene was recovered perfectly from a limited number of samples. An image of the recovered radar scene is shown in figure 4.5. Even more, figures 4.6 and 4.7 show the recovered radar scene when the matched filter is used. It can be noticed in both figures that there is a significant amount of noise generated by the matched filter. This interference has the potential of hiding objects or even making difficult to distinguish objects close together. This represents one of the major advantages of using compressive sensing to recover radar information.

Table 4.2: Parameters used to generate chaos based FM signal for moving targets

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Center Frequency</td>
<td>100 MHz</td>
</tr>
<tr>
<td>Sampling Time</td>
<td>1 ns</td>
</tr>
<tr>
<td>Duration of the Signal</td>
<td>0.2 μsec</td>
</tr>
<tr>
<td>Chaotic Regime</td>
<td>1.7</td>
</tr>
<tr>
<td>Target Velocity</td>
<td>$-6 \times 10^{-8}$ m/s to $6 \times 10^8$ m/s</td>
</tr>
</tbody>
</table>
Figure 4.4: Comparison between original and reconstructed scene using cvx

Figure 4.5: Reconstructed scene using cvx
Figure 4.6: Matched filter results

Figure 4.7: 3-D plot of matched filter results
Chapter Five

Conclusion and Future Work

In this research work, compressive sensing was proposed as a method to increase the resolution of chaotic radar. A chaos based FM signal was used as the transmitted signal. This signal was generated using a Bernoulli sequence, and it was used to create the measurement matrix. The radar scenes for stationary and non-stationary targets were created to compare the performance of this system against the matched filter output.

For stationary targets, the radar scene was a one dimensional vector, in which each position in the vector represents the target position in space. The reflected signal was generated by multiplying the measurement matrix and the radar scene together. However, in order to be able to apply compressive sensing the simulated received signal was under sampled. By using optimization techniques it was possible to recover the radar scene from the under sampled received signal. In order to compare the results from the proposed system a traditional matched filter was used to recover the radar scene from the original received signal. Even though, the matched filter was able to recover the radar scene, artifacts were introduced, making it difficult to identify and recognize a radar target. On the other hand, compressive sensing demonstrated that it was possible to recover the radar scene perfectly even when the received signal was under sampled.

For non-stationary targets, the radar scene was a two dimensional matrix, in which one direction represents the target’s range, and the other direction represents the target’s velocity. The reflected signal was generated by multiplying the measurement matrix and a vectorized version of the radar scene. In order to be able to apply compressive sensing, the simulated received signal was under sampled. By using optimization techniques it was possible to recover the radar scene from the
under sampled received vector. A traditional matched filter was used to recover the radar scene from the original received signal in order to compare the results from the proposed system. In this case, the matched filter was capable of recovering the radar scene. However, there was a considerable amount of noise introduced by the matched filter that made the identification of the target a daunting task. On the other hand, compressive sensing showed that it was possible to recover the radar scene perfectly even when the received signal was under sampled.

In conclusion, the proposed system was capable of recovering the radar scene from a limited number of samples of received signal, when chaotic radar was considered. The results were compared against the matched filter output, and it was observed that even when the matched filter could recover the radar scene, there were artifacts that made the target identification a difficult task. As a result, it can be concluded that compressive sensing can potentially increase the resolution of chaotic radar.

5.1 Future Work

Compressive radar is a new technique, and more research is required before it can be fully implemented in the real world. The following is a list of ideas regarding the future of this work.

- Apply compressive sensing to other types of chaotic signals.
- Apply compressive sensing to signals that are corrupted with noise.
- Apply compressive sensing to radar imaging systems.
References


Appendix A: Stationary Targets

%MATLAB code for simulating and reconstructing the scene using cvx for
%stationary targets
clc
close all
clear all
fc=100e6; %Center Frequency
c=3e8; %Speed of light
ts=1/(10*fc); %Sampling time
T=1e-6; %Duration of the signal
N=ceil(T/ts); %No.of samples

A1=0.5;
j=sqrt(-1);
K1=10; %Modulation Index
r1=1.7; %Chaotic Regime 1.4<r<2
M=N;
K=2*N;
I=zeros(K,M); %Initializing the size of measurement matrix

x1_0=rand(1)-0.5; %generating Bernoulli map
if x1_0<0
  x1(1)=(r1*x1_0)+0.5;
else
  x1(1)=(r1*x1_0)-0.5;
end

for i=2:M %generating the Bernoulli sequence
  if x1(i-1)<0
    x1(i)=(r1*x1(i-1))+0.5;
  else
    x1(i)=(r1*x1(i-1))-0.5;
  end
end
for n=1:N  %generating the discrete version of chaotic function
  y1=0;
  for k=1:n
    y1=x1(k)+y1;
  end
  f1(n)=y1;
end

S1=real(A1.*exp(j*2*pi*K1.*f1));  % CBFM signal

for l=1:N  %generating the measurement matrix
  for k=1:2*N
    if (k-l+1)>0 && (k-l+1)<length(S1)+1
      I(k,l)=S1(k-l+1);
    end
  end
end

t=(0:length(I(:,1)))*ts;
d=c*(0:length(I(1,:)))*ts;

figure(1)
imagesc(d,t,I)
xlabel('\bf Distance (m)')
ylabel('\bf Time (sec)')
title('\bf Measurement matrix')

% Scene Vector
X=zeros(M,1);
X(200)=0.4;
X(400)=1;
X(958)=.5;
figure(2)
stem(c*(0:length(X)-1)*ts,X)
title('f Scene Vector with three targets')
xlabel('f Distance (m)')
ylabel('f Reflectivity')

% Received signal
Z=I*X;

% Under-sampling the received signal and measurement matrix
Z_new=zeros(700,1);
I_new=zeros(700,1000);
rev=sort(randsample(1:2000, 700));
counter=1;
for m=1:2000
    if sum(rev==m)==1
        I_new(counter,:)=I(m,:);
        Z_new(counter,:)=Z(m,:);
        counter=counter+1;
    end
end
figure(3)
imagesc(I_new)
title('f Reduced set of Measurement Matrix')
xlabel('f Distance')
ylabel('f Time')
axis off

% Reconstruction of the scene vector using cvx
cvx_begin
variable w(n);
minimize(norm(w,1));
subject to
    I_new*w==Z_new;
cvx_end
figure(4)
stem(c*(0:length(w)-1)*ts,w)
title(\'\bf Reconstructed Signal using cvx\')
xlabel(\'\bf Distance (m)\')
ylabel(\'\bf Reflectivity\')

figure(5)
plot(c*(0:length(X)-1)*ts,X,'x',c*(0:length(w)-1)*ts,w,'o')
title(\'\bf Comparison between original and reconstructed scene using cvx\')
legend(\'Original\',\'cvx\')
xlabel(\'\bf Distance (m)\')
ylabel(\'\bf Reflectivity\')

% Autocorrelation of columns of the measurement matrix
for q=1:1000
    X_corr=xcorr(I(:,q),S1);
    X_corr_mat(:,q)=(X_corr);
end
y_indx=(-1999:1999);
x_indx=0:999;

figure(6)
imagesc(x_indx,y_indx,X_corr_mat)
axis([0 999 0 999])
colormap(\'gray\')
colorbar
title(\'\bf Autocorrelation matrix of the columns of measurement matrix\')
axis off

% Correlation for the Matched Filter results
MF=xcorr(Z,S1);
figure(7)
MF_l=length(MF);
convt_max=floor(MF_l/2);
Appendix A (cont.)

conv_time=-convt_max:convt_max;
plot(conv_time*ts,MF)
title('\bf Matched Filter results')
ylabel('\bf Amplitude')
xlabel('\bf Time (sec)')
Appendix B: Non-Stationary Targets

%MATLAB code for simulating and reconstructing the scene using cvx for non-stationary targets
clc
close all
clear all
c=3e8; %Speed of light in m/s
fc=100e6; %Center Frequency
ts=1/(10*fc); %Sampling time
T=20*(1/fc); %Duration of the signal
N=ceil(T/ts); %No. of samples
A1=0.5;
j=sqrt(-1);
K1=10; %Modulation Index
r1=1.7; %Chaotic Regime 1.4<r<2
M=N;
S_Z=100; %Initializing the size of Scene Matrix
V=-0.02*c:((2*0.02)/S_Z)*c:0.02*c; %Target's velocity
fd=(2*V./c)*fc; %Doppler frequency
t=(0:N-1)*ts;
d=2*c*t(1:100); %Distance Vector

x1_0=rand(1)-0.5; %generating Bernoulli map
if x1_0<0
    x1(1)=(r1*x1_0)+0.5;
else
    x1(1)=(r1*x1_0)-0.5;
end

for i=2:M %generating the Bernoulli sequence
    if x1(i-1)<0
        x1(i)=(r1*x1(i-1))+0.5;
    else
        x1(i)=(r1*x1(i-1))-0.5;
    end
%generating the discrete version of chaotic function
y1=0;
for k=1:n
    y1=x1(k)+y1;
end
f1(n)=y1;
end

% CBFM signal
for l=1:length(fd)
    S1(l,:)=real(A1.*exp(j*2*pi*K1.*f1).*exp(j.*2.*pi.*K1.*fd(l).*t));
end

%Initializing the size of measurement matrix
I=zeros((S_Z)^2,(S_Z-1)+length(S1));
counter_S1=1;
shift_I=0;
for r=1:(S_Z)^2
    I(r,(1+shift_I:200+shift_I))=S1(counter_S1,:);
    shift_I=shift_I+1;
    if mod(r,S_Z)==0
        counter_S1=counter_S1+1;
        shift_I=0;
    end
end

figure(1)
imagesc(I)
title('\textbf{Measurement Matrix}')
xlabel('\textbf{Time}')
ylabel('\textbf{Doppler}')
axis off

figure(2)
imagesc(I)
axis([0 250 0 250])
title('\bf zoomed image of Measurement matrix')
xlabel('\bf Time')
ylabel('\bf Doppler')
axis off

%Scene Matrix
XX=zeros(S_Z,S_Z);
XX(10,4)=1;
XX(13,10)=1;
XX(13,11)=1;
XX(13,20)=1;
XX(13,22)=1;
XX(50,50)=1;
XX(70,50)=1;
XX(30,80)=1;
X=reshape(XX',[],1);

figure(3)
imagesc(d,V,XX)
colormap('gray')
colorbar
title('\bf Scene with Multiple Targets')
xlabel('\bf Distance (m)')
ylabel('\bf Velocity (m/s)')

figure(4)
stem(X)
title('\bf Scene Vector')
ylabel('\bf Reflectivity')
% Received Signal
S_R=X'*I;
figure(5)
plot((0:length(S_R)-1)*ts,S_R)
title('fReceived Signal')
xlabel('fTime (sec)')
ylabel('fAmplitude')

%under-sampling the received signal and measurement matrix
S_Rnew=zeros(1,100);
I_new=zeros((S_Z)^2,100);
rev=sort(randsample(1:(200+S_Z-1),100));
counter_columns=1;
for c=1:(200+S_Z-1)
    if sum(rev==c)==1;
        I_new(:,counter_columns)=I(:,c);
        S_Rnew(:,counter_columns)=S_R(:,c);
        counter_columns=counter_columns+1;
    end
end
figure(6)
imagesc(I_new)
title('fReduced set of measurement matrix')
xlabel('fTime')
ylabel('fDoppler')
axis off

figure(7)
imagesc(I_new)
axis([0 100 0 250])
title('fzoomed in image of reduced set of Measurement matrix')
xlabel('fTime')
ylabel('fDoppler')
axis off
%reconstructing the scene using cvx
cvx_begin
variable w(S_Z^2);
minimize(norm(w,1));
subject to
w'*I_new==S_Rnew;
cvx_end

figure(8)
stem(w)
title('f Reconstructed Signal using CVX')
ylabel('f Reflectivity')

figure(9)
plot(1:length(X),X,'x',1:length(w),w,'o')
title('f Comparison between original and reconstructed scene using cvx')
legend('Original','CVX')
ylabel('f Reflectivity')

ww=reshape(w,S_Z,S_Z);
figure(10)
imagesc(d,V,ww')
colormap('gray')
colorbar
title('f Reconstructed Scene using cvx')
xlabel('f Distance (m)')
ylabel('f Velocity (m/s)')

%Matched filter results
for i=1:length(S1(:,1))
    MF(i,:)=xcorr(S_R,S1(i,:));
    MF_l=length(MF(i,:));
    convt_max=floor(MF_l/2);
Appendix B (cont.)

\[ \text{conv_time} = -\text{convt}\_\text{max}:\text{convt}\_\text{max}; \]
figure(11)
plot(conv_time,MF(i,:))
title('\textbf{Matched Filter results}')
xlabel('\textbf{Convolution Time}')
end

figure(12)
imagesc(MF)
colormap('gray')
colorbar
axis off
title('\textbf{Matched filter results}')

figure(13)
surf(MF)
title('\textbf{Surf Plot of Matched filter results}')